

所別： 電子工程研究所 組別： *** 科目： 工程數學

注意： 不准 一般 工程用計算器，考試時間總計：100 分鐘。試題共 1 頁，第 1 頁

1. Solve the initial problem of the differential equation. (15%)

$$y' = y \frac{(x-1)^2}{y^2 + 3}; y(2) = -1.$$

2. Solve the differential equation. (15%)

$$y'' - 4y' + 4y = 6e^{2x}$$

3. In the following problem, use the linearity of the Laplace transform to find the inverse Laplace transform of the function. (20%)

$$\frac{1}{s^2(s+4)^2}$$

4. Find an orthogonal matrix that diagonalizes the matrix and compute the indicated power of matrix. (20%)

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}; \mathbf{A}^{50}.$$

5. Let

$$f(t) = \begin{cases} |t| & \text{for } -1 \leq t \leq 1 \\ 0 & \text{for } t > 1 \text{ and for } t < -1 \end{cases}.$$

Find the Fourier transform of the function. (20%)

6. In the following problem, evaluate $\oint_C \mathbf{F} \cdot d\mathbf{R}$. \mathbf{R} is position vector and the symbol \oint_C means a line integral is taken around a closed curve. The curve is oriented counterclockwise. (Hint: Green's theorem) (10%)

$$\mathbf{F} = 6xy^2 \mathbf{j}, C \text{ is the circle of radius 3 about the origin.}$$